PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Underground Temperature at Oxford in the Year 1899, as Determined by Five Platinum-Resistance Thermometers

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VI. *Underground Temperature at Oxford in the Year 1899, as determined by five Platinum-resistance Thermometers.*

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[Plates 1, 2.]

Description of the Apparatus and Mode of Reduction of the Observations of Earth Temperatures.

The instruments with which the earth-temperatures given in this paper were observed, were five platinum-resistance thermometers of the Callendar and Griffiths pattern,* made by the Cambridge Scientific Instrument Company. These were purchased by the late Mr. Stone, and were placed in position under his direction shortly before his death.

The method of platinum thermometry seemed to be particularly suitable for this class of work, on account of the immunity it enjoys from certain errors attending the use of the long-stemmed mercurial or spirit thermometers ordinarily employed for underground temperatures.

A higher degree of accuracy might, therefore, reasonably be expected, and the discussion which follows of the first complete year's observations at the Radcliffe Observatory shows, I think, that this anticipation has been justified. Some discrepancies between theory and observation no doubt appear, but they are of a character which seems to indicate a difference between the assumptions on which the theory is based and the conditions actually prevailing in the stratum of gravel in which the thermometers are buried, rather than thermometric errors affecting the observations themselves.

The thermometers are inserted in undisturbed gravel, the first four lying one under the other, in a vertical plane beneath the grass of the south lawn, and within a few feet of the Stevenson screen in which the dry and wet bulb, and the maximum and minimum, thermometers are suspended.

In order that the thermometers might lie in practically unbroken ground, the following method of placing them was adopted. A pit was dug at the edge of the

* See the Cambridge Scientific Instrument Company's "Descriptive List of Instruments," page 20.

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grass about 5 feet long by 4 feet wide. One edge of the pit coincided with the edge of the grass plot, and the corresponding side of the pit was made as nearly vertical as possible. Into this vertical face four iron tubes were driven horizontally, the tubes being formed with spikes at their ends to facilitate this operation. The tubes are 4 feet long, and into them the thermometers were inserted with the leads attached, the mouths of the tubes were sealed up with tow and red lead, and the pit filled in.

The first four thermometers were placed at depths of (approximately) 6 inches, 1 foot 6 inches, 3 feet 6 inches, and 6 feet respectively; but Mr. STONE soon saw the advisability of placing another at a lower level, and intended to have gone to a depth of 20 feet. But as water was met with at a depth of 10 feet 6 inches, he decided to place it just above the water level, at a depth of 10 feet.

This thermometer was buried, not directly under the four earlier ones, but in a separate pit at the other side of the Stevenson screen. This was apparently done to avoid disturbing the leads of the thermometers which were already in position, but it would have been rather more satisfactory if all had been placed in the same vertical plane.

It is also, perhaps, to be regretted that one or two similar thermometers were not buried to considerably greater depths. The presence of water, however, complicated matters and introduced conditions different from those which prevailed in the dry gravel above. It is not, for example, to be supposed that the thermal conductivity or the diffusivity of permanently water-logged gravel would be the same as that of the drier material above it. Hence it would appear necessary to put at least two thermometers below the permanent water-level in order to study the flow of heat under such circumstances. Besides, it is highly probable that the gravel stratum is not very much thicker than 10 feet. Excavations in the neighbourhood show that the blue Oxford clay is likely to be met with at any depth below 12 feet from the surface, and in this, of course, the thermal conditions would be likely to prove wholly different from those in the gravel.

The actual depths of the various thermometers as measured in October, 1898 (when the pits were standing open to enable us to re-standardise the thermometers) were as follows:—

Thermometer .	1	2	3	4	5
Depth	$6\frac{1}{2}$ in.	1 ft. 6 in.	3 ft. $6\frac{1}{2}$ in.	5 ft. $8\frac{1}{2}$ in.	9 ft. $11\frac{1}{2}$ in.

These thermometers, with the Callendar and Griffiths resistance box, which could be connected with each thermometer through a switchboard, had been set up as I have stated, shortly before Mr. STONE's death.

On my appointment to the post of Radcliffe Observer, I took an early opportunity of examining the apparatus, and partly with a view of familiarising myself with all its details, I proceeded to determine the comparative values of the coils, and to re-standardise a spare thermometer which was kept in the observing room for general purposes.

This examination led to the discovery of discrepancies in the readings of the apparatus which troubled me for a long time, and which necessitated a large number of experiments extending at intervals over the greater part of a year before they were traced to their sources and eliminated.

In this part of the work I have to acknowledge the very generous help and advice of Mr. E. H. GRIFFITHS, F.R.S., who was kind enough to come to Oxford on more than one occasion to place his experience at our disposal, and who, at one stage of the investigation, took the resistance box and spare thermometer to Cambridge to subject them to a prolonged examination in his own laboratory.

These discrepancies, though serious in view of the accuracy which we had reason to expect from the apparatus, were still small quantities confined within one or two tenths of a centigrade degree. They were, for the most part, traced eventually to uncertainties in the contacts at the switchboard, and a want of perfect insulation in the older leads. These consisted of four india-rubber covered wires which, in the underground portion, passed through leaden pipes, but within the observing room were without the leaden covering. It was found that these were very susceptible to damp, and that the insulation fell away very rapidly when there was much moisture in the air, thus giving rise to very puzzling and troublesome discrepancies.

In September, 1898, the switchboard was improved and new composition cable leads substituted, which extended without interruption from the thermometers right up to the switchboard. Since these changes were effected the discrepancies have ceased to appear, except on one occasion (viz., October 27, 1899), when it was found that the short flexible lead from the switchboard to the resistance box was thoroughly damp. On lighting a fire in the observing room to dry the covering of this lead, the irregularities disappeared.

Since that date up till the end of March of this year (1900) I have kept a gas light burning continuously in the room, to prevent the deposition of moisture, and have experienced no further trouble of the sort.

The resistance box is in its general design similar to that described by Mr. GRIFFITHS,* but simplified to suit the particular class of work for which it was intended.

It is provided with three principal coils A, B, C, whose nominal values are 20, 40 and 80 box units respectively, a box unit being about 0·01 ohm. There are two additional coils, one for the calibration of the bridge wire, and another, which we have called the "concealed coil," whose value is about 240 box units, which was inserted for convenience to balance approximately the resistance of the thermometers at 0° C. when the coil A was also in the circuit, so that the reading of the bridge wire under these circumstances might be as nearly zero as possible.

* 'Nature,' November 14, 1895.

On account of the differential character of the equation (a), (p. 242), the value of this coil does not concern us except in computing the correction for the temperature of the box, and then an approximate value only is required.

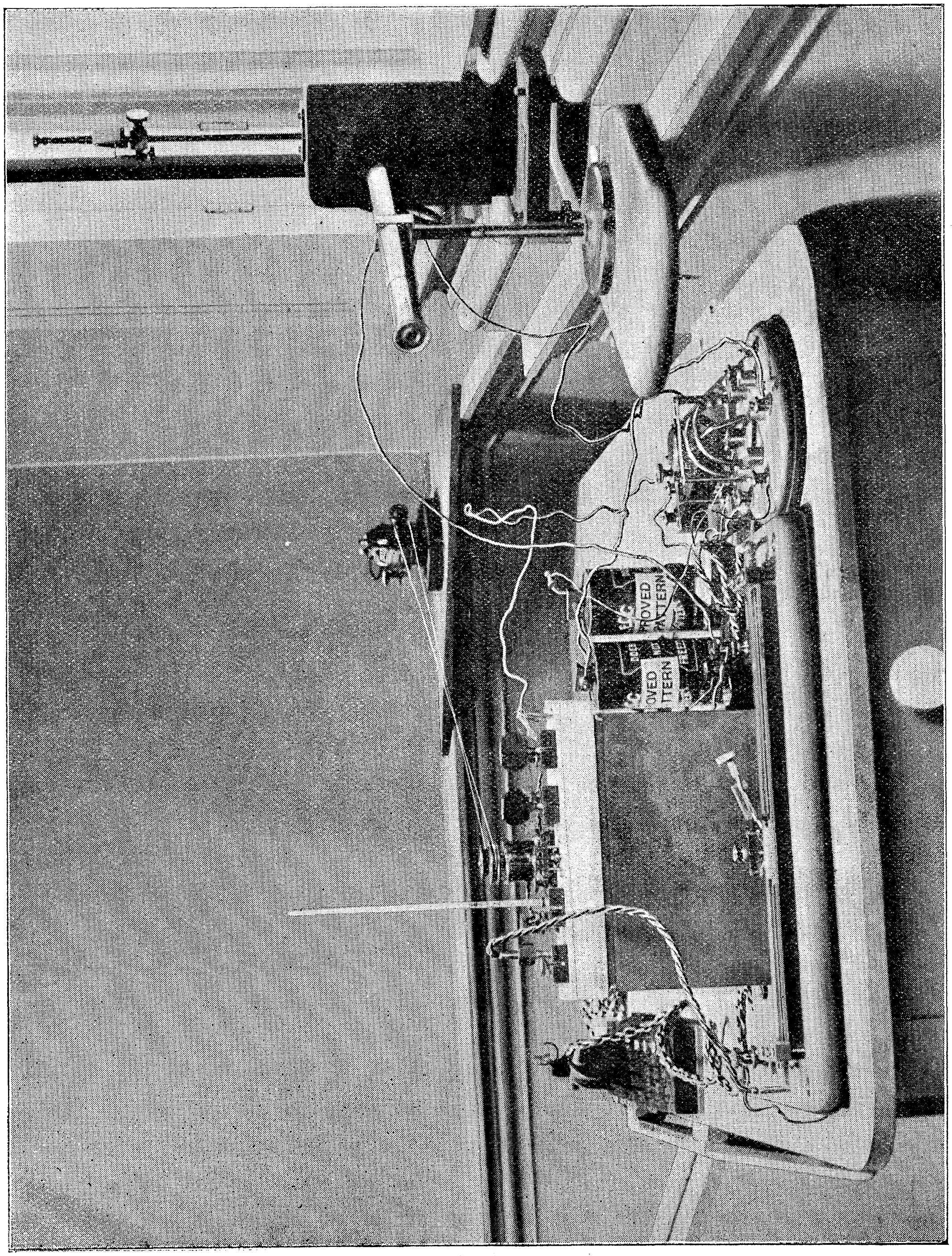

Fig. 1.—Resistance Box and Galvanometer at the Radcliffe Observatory, Oxford.

The apparatus is provided with a slow motion contact-maker of Mr. HORACE DARWIN's pattern,* and GRIFFITHS' thermo-electric key.†

* 'Nature, November 14, 1895.

† 'Phil. Trans.,' A, vol. 184 (1893), pp. 397–8.

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The galvanometer microscope is placed on a window ledge to the right of the resistance box, in such a position that the observer can manipulate the commutator for reversing the direction of the current without removing his eye from the eye-piece.

The general arrangement of the apparatus is shown in fig. 1. To the right is the galvanometer and microscope; underneath in front is the commutator, and behind it the contact key. On the extreme left is the switchboard, and in the corner of the room is seen a small electric motor for stirring the oil in which the resistance coils are immersed.

In the standardisation of the apparatus the method described by Mr. GRIFFITHS in 'Nature,' of November 14, 1895, was in the main followed. The temperature coefficient was determined by Mr. GRIFFITHS when the apparatus was under examination in his own laboratory at Cambridge. Two separate determinations made in 1898 (July 27 and August 8) gave the following results:—

Date.	Range of Temp.	Temp. Coeff.
July 27 . .	9°·18	0·000242
August 8 . .	12 ·51	0·000240

In the reduction of the observations, the value 0·00024 has been adopted. The accuracy of this value has been borne out by subsequent observations in several different ways. Thus, for example, the invariable steadiness in the changes of No. 5, whatever might be the temperature of the box, indicated a high degree of accuracy in the adopted value of the constant.

For the determination of the coil values and the unit of the bridge wire scale, the following observations were made at the Radcliffe Observatory:—

	C − B − A =	B − A =	A =
	20·051	19·851	19·603
	20·046	19·849	·600
	20·043	19·848	·601
	—	19·853	·602
	—	19·847	—
Means . .	20·047	19·850	19·601

From these we obtain, as in Mr. GRIFFITHS' paper referred to above,

$$\left.\begin{aligned} C &= 80{\cdot}158 \\ B &= 39{\cdot}979 \\ A &= 19{\cdot}863 \end{aligned}\right\} \text{mean box units.}$$

and one scale division of the bridge wire is equal to

1·0134 mean box units.

We thus get the following table giving the correction for the particular arrangement of coils in use, in which the corrections have each been diminished by 20 simply for the convenience of having the nominal readings of the box approximately equal to the temperature of the thermometers :—

TABLE I.—Correction for Coils.

Plugs in	Correction.	Coils in Circuit.
A, B, C	− 20·000	None
B, C	− 0·137	A
A, C	+ 19·979	B
C	+ 39·842	A, B
A, B	+ 60·158	C
B	+ 80·021	A, C
A	+100·137	B, C
None	+120·000	A, B, C

The correction to the bridge wire scale is also deduced from the following table, the correction being always of the same sign as the reading of the scale :—

TABLE II.—Bridge Wire Table.

R.	Corr.	R.	Corr.	R.	Corr.
0	0·000	6	0·080	12	0·161
1	·013	7	·094	13	·174
2	·027	8	·107	14	·188
3	·040	9	·121	15	·201
4	·054	10	·134	16	·214
5	·067	11	·147	17	·228
6	·080	12	·161	18	·241

We have next to correct for the temperature of the coils and bridge wire. This temperature is read from a mercury thermometer which stands in the oil in which the coils are immersed, its stem protruding through the marble slab which forms the top of the box. The oil in the box was stirred before each series of readings by means of a small electric motor. When the observations were prolonged for any considerable time, as in the process of standardising, the oil was stirred at frequent intervals.

All observations have been reduced to a standard temperature of 14° C., as being about the mean temperature of the observing room throughout the year.

If R_θ denotes the observed resistance at any temperature θ, and R_{14} the corresponding resistance for a temperature of the coils of 14° C., they are connected by the relation,

$$R_{14} = R_\theta\{1 + k(\theta - 14°)\}.$$

or adopting the temperature coefficient, 0·00024, as determined by Mr. GRIFFITHS,

$$R_{14} - R_\theta = R_\theta \times 0{\cdot}00024 \times (\theta - 14°).$$

In this expression R_θ is the total resistance in the circuit, and since this includes the resistance of the "concealed coil," we require to know approximately the value of that coil in the right-hand side of the equation. This is, perhaps, most easily determined from the observations of the thermometers themselves at 100° C. and 0° C., combined with the constant value found by Mr. GRIFFITHS for the ratio of the corresponding resistances R_1 and R_0.

Thus, if X be the value of this coil, r_0 that of the other coils in use and the bridge wire when the thermometer is packed in melting ice, and r_1 that of the coils and bridge wire when it is immersed in steam, reduced to mean box units at 14° C., then the total resistances in the two cases are, $X + r_1$ and $X + r_0$, and if we take the ratio of these resistances to be 1·3872,* as found by Mr. GRIFFITHS for the wire used in the construction of this instrument, then $\frac{X + r_1}{X + r_0} = 1{\cdot}3872$, and therefore $X = \frac{r_1 - 1{\cdot}3872 r_0}{0{\cdot}3872}$.

The values of X found in this way from the observations made on October 4, 5, and 6, 1898, for the purpose of standardising the thermometers, are as follows:—

Thermometer.	X.
1	240·65
2	·65
3	·77
4	·77
5	·60
A	·65
Mean	240·68

For any arrangement of coils (Y) and any bridge wire reading (R) we have therefore the total resistance in the circuit, X + Y + R, and the coefficient of $(\theta - 14°)$ in the correction for temperature is

$$(X + Y + R) \times 0{\cdot}00024.$$

We thus find the following table for the two different arrangements which have been used in the observations:—

* 'Nature,' November 14, 1895, p. 45.

TABLE III.—Coefficient of $(\theta-14°)$.

Plugs in	B+C	A+C	Plugs in	B+C	A+C	Plugs in	B+C	A+C
Bridge wire reading.			Bridge wire reading.			Bridge wire reading.		
−18	0·0581	0·0629	−5	0·0612	0·0660	+8	0·0643	0·0691
17	83	31	4	14	62	9	46	94
16	86	34	3	17	65	10	48	96
15	88	36	2	19	67	11	50	·0698
14	90	38	−1	22	70	12	53	·0701
13	93	41	0	24	72	13	55	03
12	95	43	+1	26	74	14	58	06
11	·0598	46	2	29	77	15	60	08
10	·0600	48	3	31	79	16	62	10
9	02	50	4	34	82	17	65	13
8	05	53	5	36	84	+18	·0667	·0715
7	07	55	6	38	86			
−6	·0610	·0658	+7	·0641	·0689			

The temperature on the platinum scale corresponding to a resistance, R, is deduced from CALLENDAR'S formula,

$$pt = 100\ (R-R_0)/(R_1-R_0)^* \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (a.)$$

where R_1 is the resistance at 100° C., and R_0 at 0° C., each of them being reduced to mean box units at the standard temperature (14°).

From a very careful series of observations made at the Radcliffe Observatory on October 4 and 6, 1898, the following separate values of the zero points were obtained, each value being the mean of several closely accordant settings :—

Zero Points of the Thermometers.

Thermometer .	1.	2.	3.	4.	5.
1898, October 4	0·306	0·420	0·481	0·323	—
" 6	·306	·432	·496	·338	0·239
" 6	·305	·434	·498	·333	·239
Adopted means	0·31	0·43	0·49	0·33	0·24

The temperature of steam was observed on October 4 and 5, with the following results for the several thermometers :—

* 'Phil. Trans.,' A, vol. 178, p. 195.

Temperature of Steam.

Thermometer .	1.	2.	3.	4.	5.
1898, October 4	—	—	101·610	101·389	—
„ 5	101·286	101·474	·589	·367	101·179
Adopted values	101·29	101·47	101·60	101·38	101·18

To those who have standardised naked platinum thermometers the discrepancies in the separate results for the b. p.'s of Nos. 3 and 4 may appear large. It should, however, be pointed out that it was necessary to standardise these instruments while they were sealed up in strong brass tubes with heavy leaden-covered leads attached; hence it was impossible to eliminate altogether the effects of conduction along these tubes, but for reasons given on p. 244 it was not considered necessary to take further precaution against the small errors arising from this cause. In determining the zero points the thermometers were placed in a trough 3 feet long, and any error arising from this cause was very much diminished.

One of the most important considerations in connection with this subject is the degree of permanence in the fundamental points as determined at considerable intervals of time. It was the occurrence of discrepancies between the values which I found in my first observations and those determined about a year and a half previously at the time that the instruments were set up, which induced me to have all the thermometers exhumed and to make a thorough re-examination of the whole apparatus. This examination led eventually to my discarding the original leads, the insulation of which was found to fall off very much when they became damp.

Another series of discrepancies was traced to an uncertainty in the contacts at the switchboard. In the original form the four steel prongs in which the fourfold lead from the resistance box terminates, were inserted into mercury cups into which also were led the four brass strips to which the thermometer leads were soldered. By having the steel prongs amalgamated, and adding springs to keep each prong firmly pressed against the brass strips immersed in the mercury, a great improvement was experienced; and, since this change was effected, we have had no trouble from the same cause. It has been the habit, too, to make the observations from time to time with the four steel prongs in both positions, which affords a very satisfactory check on the character of the contacts.

Taking advantage of a visit from Mr. GRIFFITHS on October 6, 1899, I had thermometer No. 1 (6 inches) dug up, and we examined its zero point after exactly a year's continuous observations. Determined in the same way as in the previous year, the zero point of this thermometer was found to agree with the earlier value to less than 0°·005 C., the actual values being

In 1898 . . . . 0·306
and in 1899 . . . . 0·302

During the observations of 1898, the temperature of the box ranged from 15°·81 to 16°·05, whilst in 1899 it stood at 10°·36. The close agreement of these two results, therefore, taken at such an interval of time and at temperatures differing so considerably—by about one-sixth of the whole range with which we are concerned—affords a further confirmation of the accuracy of the adopted value of the temperature coefficient and the general consistency of the apparatus.

It is clear that in a series of observations, such as we are at present considering, in which the highest temperature does not exceed 25° C., any change in the zero point will have considerably more influence on our results than a corresponding variation in the boiling point. Also the researches of Professor CALLENDAR, Mr. GRIFFITHS, Messrs. HEYCOCK and NEVILLE, and others, show that the change in the fundamental interval, even when the instrument is exposed to enormous variations of temperature, is very slight.

Our experience with a sixth thermometer (A), which is kept in the observing room, points in the same direction. In this instrument the fundamental interval remained practically unaltered after the lapse of a year, the actual values found for it being

In 1898 . . . . . . 101·067
and in 1899 . . . . . . 101·059

On substituting the values found above for R_0 and R_1 in formula (a) we obtain the following expressions for the separate thermometers, giving the temperature on the platinum scale corresponding to any reading R :—

Thermometer 1. $pt = (R - 0{\cdot}31)/1{\cdot}0098 = R - (0{\cdot}0097\,R + 0{\cdot}31)$
2. $pt = (R - 0{\cdot}43)/1{\cdot}0104 = R - (0{\cdot}0103\,R + 0{\cdot}43)$
3. $pt = (R - 0{\cdot}49)/1{\cdot}0111 = R - (0{\cdot}0110\,R + 0{\cdot}48)$
4. $pt = (R - 0{\cdot}33)/1{\cdot}0105 = R - (0{\cdot}0104\,R + 0{\cdot}33)$
5. $pt = (R - 0{\cdot}24)/1{\cdot}0094 = R - (0{\cdot}0093\,R + 0{\cdot}24)$

The expressions in brackets on the right-hand side are the corrections which must be applied to R to obtain the temperature on the platinum scale. They may be tabulated in a very simple form for each thermometer, so that the platinum temperature can be at once deduced from the reading of the resistance.

TABLE IV.—Reduction to Platinum Scale.

Thermometer .	1.	2.	3.	4.	5.
R = −10 . .	−0·21	−0·33	−0·37	−0·23	−0·15
0 . .	−0·31	−0·43	−0·48	−0·33	−0·24
+10 . .	−0·41	−0·53	−0·59	−0·43	−0·33
+20 . .	−0·50	−0·64	−0·70	−0·54	−0·43
+30 . .	−0·60	−0·74	−0·81	−0·64	−0·52

A complete determination of temperature on the platinum scale by means of one of the sunken thermometers is, therefore, reduced to the following simple steps :—

(1) The balancing of the galvanometer and reading of the bridge wire scale (R) and the temperature (θ) of the box.

(2) To R is to be added the correction for the particular arrangement of coils used, from Table I.

(3) The correction to reduce the bridge wire reading to mean box units, from Table II.

(4) The reduction to standard temperature (14°). The quantity taken from Table III. multiplied by (θ − 14) gives this correction.

(5) The correction from Table IV.

It only remains to reduce the temperature thus expressed from the platinum to the air scale.

The relation connecting these two, established by Professor CALLENDAR,* is

$$d = t - pt = \delta\left\{\left(\frac{t}{100}\right)^2 - \frac{t}{100}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

in which pt is the platinum temperature, t the temperature on the air scale, and δ a constant.

For a completely independent standardisation it would be necessary to determine the resistance at some third known temperature in order to obtain the value of δ, but the experiments of CALLENDAR and GRIFFITHS have shown that although the value of δ varies from one specimen of platinum to another, it is a constant for any particular sample of wire. References to the original papers bearing on this point are given in Mr. GRIFFITHS' article in 'Nature' cited above.

The value of δ for the particular wire used in the Oxford instrument was determined at Cambridge to be 1·512.† If it were intended to employ the Oxford apparatus for the determination of temperatures over a very wide range, it would doubtless have been advisable to make an independent determination of the value of this constant. Since, however, the range −15° C. to +25° C. will cover all the variations of earth temperatures with which alone we are here concerned, and since within that range the correction does not amount to as much as 0·3, an error of even 0·050 (which is quite inadmissible) in the value of δ would not affect our results.

Writing $pt + d$ for t in equation (b), and remarking that since $d/100$ is less than 0·003, its square may be neglected, we find

$$d = \delta(\tau^2 - \tau)/\{1 + (1 - 2\tau)\delta/100\}$$

τ being written for $pt/100$.

* 'Phil. Trans.,' A, 1887.

† *Cf.* the Report of the Committee of the British Association for improving the Construction of Practical Standards for use in Electrical Measurements. Bradford, 1900. [September 16, 1900.]

We thus obtain the following table for the correction from the platinum to the air scale, for every degree of the former from $-15°$ to $+25°$.

TABLE V.—Reduction from the Platinum to the Air Scale.

pt.	Corr. to air.	*pt.*	Corr. to air.	*pt.*	Corr. to air.	*pt.*	Corr. to air.
°	°	°	°	°	°	°	°
−15	+0·256	−5	+0·078	+5	−0·071	+15	−0·191
14	·237	4	·062	6	·084	16	·201
13	·218	3	·046	7	·097	17	·211
12	·199	2	·030	8	·110	18	·221
11	·181	−1	+ ·015	9	·122	19	·231
10	·163	0	·000	10	·134	20	·240
9	·146	+1	− ·015	11	·146	21	·249
8	·128	2	·029	12	·158	22	·257
7	·111	3	·043	13	·169	23	·266
6	·095	4	·057	14	·180	24	·274
− 5	+ ·078	+5	− ·071	+15	− ·191	+25	− ·281

The reduction of the observations is thus of a very simple character; but it may be still further simplified, and the chance of arithmetical errors occurring in individual cases greatly reduced, if not wholly removed, by the preparation of a table giving the total correction to the bridge wire reading for each arrangement of coils, the arguments in each case being the bridge wire reading and the temperature of the box. Of course, if a great variety of coils were in use, such tables would attain dimensions out of proportion to their usefulness, as it would be necessary to construct a special table for each separate combination of coils. But in observations of underground temperature, the range of the readings of any particular thermometer is comparatively limited, so that only two different arrangements of the plugs are necessary, coil A serving for about eight months of the year, and coil B coming into use for about four months in summer.

It is thus a simple matter to compute tables that will cover all cases. Tables of this sort were prepared for each of the earth thermometers, from which the correction to the bridge wire reading, to reduce to the corresponding temperature on the air scale, could be obtained at one step by simple interpolation.

For some little time before and after the epochs at which the change from one coil to the other is made (about the end of May and the end of September) it is possible to read the thermometers with either arrangement, and thus a check can be imposed upon the general performance of the apparatus.

In 1899 the changes were made on June 1 and September 26, and the following readings of the 10-foot thermometer, in which the changes are very slow and regular, afford very satisfactory evidence of the consistency of the readings.

May 28	9°·55 C.	Coil A.	Sept. 22	14°·42 C.	Coil B.
29	9·59		23	14·41	
30	9·63		24	14·41	
31	9·66		25	14·40	
June 1	9·70	Coil B.	26	14·39	Coil A.
2	9·75		27	14·37	
3	9·80		28	14·35	
4	9·82		29	14·34	

Discussion of the Observations.

The first step in the discussion of the observations is to group them into monthly means, and thence to deduce the harmonic expressions which will represent the readings of each thermometer throughout the year.*

In this part of the work I have adopted the Fahrenheit scale, as the observations had already been reduced to this scale for comparison with our other meteorological results, and as the observations of the same kind at Greenwich† and Edinburgh‡ discussed by Professor EVERETT are expressed in the same scale, there seemed to be a distinct advantage in retaining it.

On account of the inequality in the lengths of the calendar months I have discarded them altogether, and, as far as possible, have divided the year into twelve portions which are alternately thirty and thirty-one days in length. As the observations are taken only once a day, it is of course necessary to have an integer number of days in each division, but the following scheme makes the differences in their lengths as small as possible, and with one exception, that of January, alternately thirty and thirty-one days. In Leap Year this exception would be removed by intercalating the extra day in January, instead of February.

* Professor W. THOMSON, "On the Reduction of Observations of Underground Temperature," 'Trans. Roy. Soc. Edin.,' vol. 22, p. 409.

† 'Greenwich Observations,' 1860 (cxciii.).

‡ Professor EVERETT, 'Trans. Roy. Soc. Edin.,' vol. 22, p. 429.

Divisions of the Year.

Ordinary Years.			Leap Year.	
Division.		No. of days.		No. of days.
I.	Jan. 1 to Jan. 30 (inclusive)	30	Jan. 1 to Jan. 31	31
II.	Jan. 31 to Mar. 1 ,,	30	Feb. 1 ,, Mar. 1	30
III.	Mar. 2 ,, April 1 ,,	31		31
IV.	April 2 ,, May 1 ,,	30		30
V.	May 2 ,, June 1 ,,	31		31
VI.	June 2 ,, July 1 ,,	30		30
VII.	July 2 ,, Aug. 1 ,,	31		31
VIII.	Aug. 2 ,, Aug. 31 ,,	30		30
IX.	Sep. 1 ,, Oct. 1 ,,	31		31
X.	Oct. 2 ,, Oct. 31 ,,	30		30
XI.	Nov. 1 ,, Dec. 1 ,,	31		31
XII.	Dec. 1 ,, Dec. 31 ,,	30		30

For the sake of convenience, I have retained below the usual names of the months for these twelve divisions of the year. A good deal might, I think, be said in favour of adopting these intervals for all meteorological returns where the means of daily observations are taken.

The monthly means so obtained for the five thermometers are given in the following table:—

Mean Monthly Temperature of the Ground at the Radcliffe Observatory, Oxford, 1899.

Thermometer.	1	2	3	4	5
Depth	6½ in.	1 ft. 6 in.	3 ft. 6½ in.	5 ft. 8½ in.	9 ft. 11½ in.
January	40°·47	42°·07	44°·68	46°·80	49°·97
February	40·09	41·34	43·25	45·08	48·33
March	41·34	41·91	43·21	44·74	47·42
April	48·77	47·66	46·61	46·40	47·37
May	54·86	52·95	50·96	49·54	48·53
June	66·73	62·89	58·29	54·73	50·88
July	69·43	65·93	62·15	58·74	53·85
August	69·23	67·79	64·88	61·66	56·39
September	59·34	61·12	62·03	61·19	57·80
October	48·99	51·14	54·29	56·12	56·71
November	46·73	48·43	50·99	52·69	54·48
December	38·14	41·08	45·35	48·58	52·33

The observations were taken within a short time of noon on every day throughout the year without exception. They were for the most part made by Mr.

McClellan in the ordinary routine, but during his vacation, or on Sundays, Mr. Wickham or Mr. Robinson took his place. All three are observers of skill and experience, and, as the results seem to show, the observations are of a remarkable degree of precision.

These means were deduced from the observations as directly obtained without any modification or correction, other than those taken from the tables referred to above, except on one date—October 27—when, as I have mentioned, the short flexible lead was found to be affected by the dampness of the air in the observing room.

This was indicated by a sudden change of about 0°·13 F. in the reading of the 10 feet thermometer, which, under ordinary circumstances changes so slowly and steadily, that its reading on any day might be predicted with certainty to within one-twentieth of a degree from the readings of two or three days preceding. On drying the lead, however, the abnormal readings disappeared by the next day, and the subsequent readings of this thermometer were found to lie along the same curve as before the discrepancy had arisen.

As the dampness of the lead disturbed only the reading of the resistance box and in no way affected the thermometers themselves, we were, therefore, able to take an interpolated value for the reading of No. 5 as a standard of comparison, and the difference between this and the actually-observed readings, viz. : 0°·13, was added as a correction to all observations made on that day.

This particular case illustrates very well the protection which the readings of a deep sunk thermometer afford against sudden changes occurring unobserved in the apparatus.

The monthly means are graphically represented in fig. 2 ; the daily readings for two periods of two months each are exhibited in Plates 1 and 2. These two periods have been selected, as the first includes the minimum and the second the maximum of the 10 feet thermometer, and both illustrate very well the steadiness of the changes in the indications of this instrument, and exhibit also the manner in which both the amplitude of a wave is diminished, and its phase retarded in passing from one thermometer to the one below it.

In fig. 3 are given the mean monthly temperature gradients beneath the surface, deduced from the same figures.

The harmonic expression to represent the temperature of any particular thermometer throughout the year will be

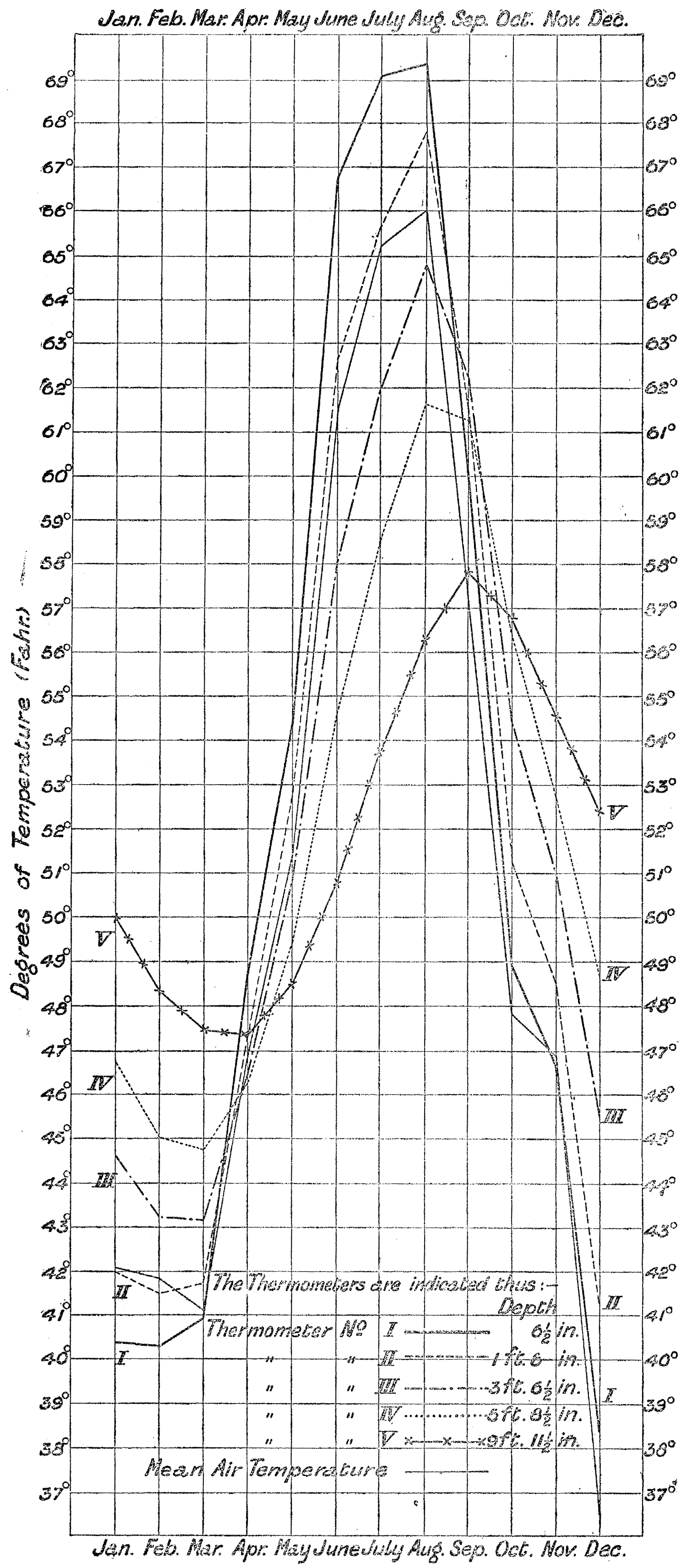
$$\theta = a_0 + a_1 \cos \lambda t + a_2 \cos 2\lambda t + \&c. \\ + b_1 \sin \lambda t + b_2 \sin 2\lambda t + \&c. \quad \ldots \quad (c)$$

or

$$\theta = a_0 + \mathrm{P}_1 \sin (\lambda t + \mathrm{E}_1) + \mathrm{P}_2 \sin (2\lambda t + \mathrm{E}_2) + \&c. \quad \ldots \quad (d)$$

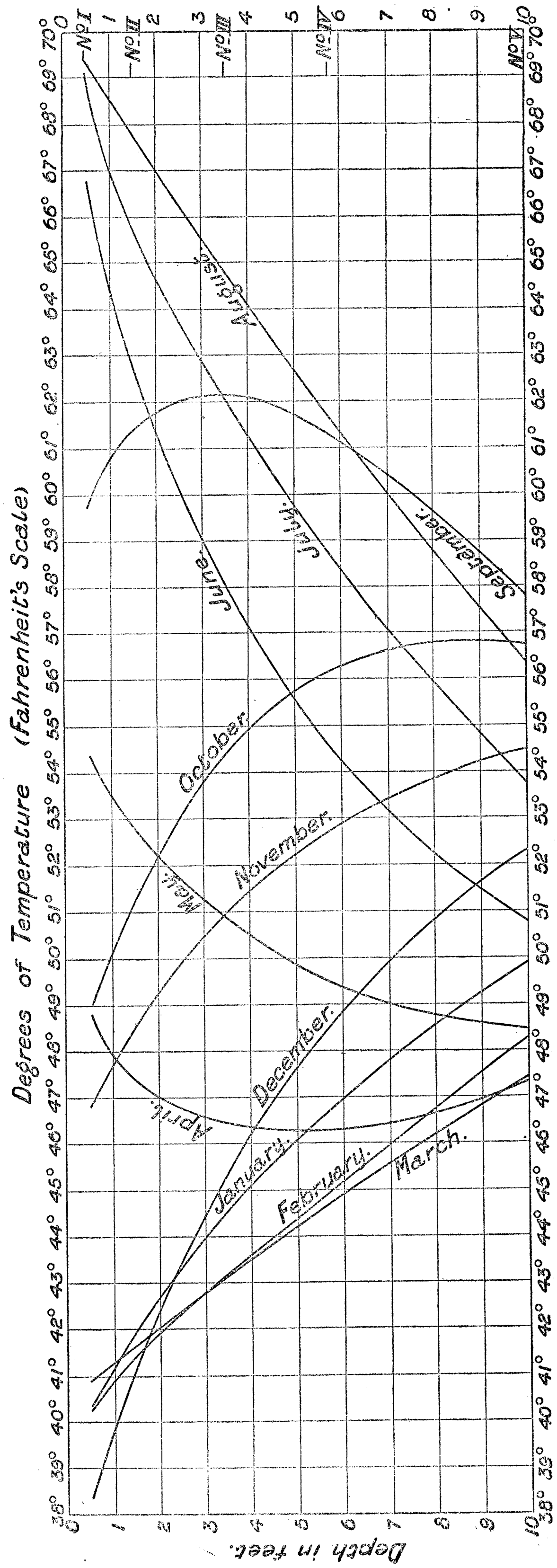
where t denotes the time represented as the fraction of a year, and λ is equal to 2π.

Fig. 2.—Mean Monthly Temperature of the Ground, 1899.



PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Fig. 3.—Mean Monthly Temperature Gradients.



NOTE:- The positions of the Thermometers are indicated by the Roman Numerals at the right of the diagram.

From the monthly means given above, we deduce the following :—

VALUES of the Coefficients.

No.	a_0	a_1	b_1	a_2	b_2	P_1	E_1	P_2	E_2
	°	°	°	°	°	°	° ′	°	° ′
5	52·005	− 1·970	−4·706	−0·068	+0·537	5·102	202 42·8	0·541	352 45·1
4	52·189	− 6·379	−5·318	+0·661	+1·029	8·305	230 10·9	1·223	32 42·1
3	52·224	− 9·467	−4·725	+1·370	+1·123	10·581	243 28·6	1·771	50 39·5
2	52·013	−12·932	−3·128	+2·130	+0·976	13·305	256 24·3	2·343	65 23·3
1	52·010	−15·337	−1·507	+3·017	+0·511	15·411	264 23·2	3·060	80 23·2
Air	50·396	−12·776	−1·621	+2·817	+1·410	12·878	262 46·1	3·177	63 39·2

In this table I have added, for comparison, the constants of the Fourier series representing the mean temperature of the air in the Stevenson screen at a height of 4 feet above the surface, deduced, as has been the custom for many years at this Observatory, from observations of a standard mercury thermometer at 8 A.M., noon, and 8 P.M.

Some interesting results appear at once from this table. As was to be expected the mean temperatures (denoted by a_0) of the soil at all depth exceeds that of the air, although the differences are less than those deduced by Professor EVERETT from the Greenwich results for thirteen years.* It will also be noticed that the annual range of temperatures for Nos. 1 and 2 exceeds that of the air. This is only true of the monthly means. The range of the mean *diurnal* air temperature is considerably greater than that observed for any of the underground thermometers. The maxima and minima are given in the following table :—

Thermometer.	Maxima.	Minima.
	°	°
Air	75·4	21·1
1	75·29	33·48
2	70·03	36·97
3	65·28	41·45
4	62·28	44·38
5	57·96	47·08

If the actual maximum and minimum for the air had been taken instead of the daily mean, the range would have been even greater.

I have not taken into account the terms depending on $3t$. For No. 5 this term is

$$0^\circ{\cdot}256 \sin(3\lambda t + 117^\circ\ 28'),$$

and in the case of the other thermometers they attain slightly larger dimensions;

* 'Greenwich Observations,' 1860 (cxciii.).

but as we are here dealing with the observations of a single year, it would be unsafe to attach much importance to results deducible from the smaller terms.

The accuracy with which the observations are represented by the first three terms of the formulæ is shown by the following table, which contains the differences between the mean monthly temperatures as computed and those actually observed :—

Comparison of Computed and Observed Mean Monthly Temperatures, C—O.

Thermometer.	1	2	3	4	5
January	−0°·78	−0°·86	−0°·55	−0°·32	0°·00
February	−0·17	−0·18	+0·07	+0·15	+0·05
March	+0·65	+0·71	+0·48	+0·22	+0·02
April	−1·28	−0·70	−0·48	−0·19	0·00
May	+1·56	+0·91	+0·25	+0·01	−0·05
June	−1·13	−1·02	−0·52	−0·24	−0·02
July	+0·93	+1·15	+0·91	+0·49	+0·06
August	−1·23	−1·10	−0·44	−0·07	+0·10
September	+0·58	−0·15	−0·69	−0·65	−0·23
October	+1·51	+1·87	+1·29	+0·73	+0·07
November	−3·03	−2·14	−1·07	−0·31	+0·18
December	+2·41	+1·52	+0·75	+0·18	−0·18

It thus appears that while the three deeper thermometers are fairly well represented by the formulæ, there are considerable differences in the two upper ones. This, especially in the case of No. 1, is largely due to the diurnal variations which make themselves felt to a depth of about 3 feet.

The surface of the ground in the neighbourhood of the spot where the thermometers are sunk, being approximately level and the gravel being, as far as we know, of a fairly uniform character for a considerable distance in all directions, the flow of heat will be represented by FOURIER's equation

$$\kappa \, d^2\theta/dx^2 = d\theta/dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (e)$$

in which κ denotes the diffusivity of the gravel, and x denotes the depth of a thermometer below the surface.

A solution of this is

$$\theta = \Sigma \mathrm{A}_n e^{-\alpha_n x} \sin (n\lambda t + \beta_n x + \gamma)$$

if

$$\alpha_n^2 - \beta_n^2 = 0 \quad \text{and} \quad 2\alpha_n\beta_n\kappa = -n\lambda.$$

Hence

$$\alpha_n = \sqrt{\frac{n\pi}{\kappa}} = -\beta_n.$$

Comparing this expression with the series (d) given on page 249 we have

$$\mathrm{P}_n = \mathrm{A}_n e^{-\alpha_n x} \quad \text{and} \quad \mathrm{E}_n = \beta_n x + \gamma.$$

For a second thermometer we have similarly

$$P_n' = A_n e^{-\alpha_n x'} \quad \text{and} \quad E_n' = \beta_n x' + \gamma,$$

and, therefore,

$$\left.\begin{aligned} \sqrt{\frac{n\pi}{\kappa}} &= \alpha_n = \frac{\log P_n - \log P_n'}{x' - x} \\ \text{and} \qquad \sqrt{\frac{n\pi}{\kappa}} &= -\beta_n = \frac{E_n - E_n'}{x' - x} \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (f).$$

Thus from each wave as observed at any pair of thermometers, we obtain two determinations of the value of κ, one from the diminution of amplitude, and the other from the retardation of phase.

In computing the value of $\sqrt{\pi/\kappa}$ I have expressed the depths in Paris feet, in order to bring out the results in terms of the same units as those employed by Professor EVERETT in his Reduction of the Greenwich Observations,* and those of Lord KELVIN already referred to. The results are given in the following tables :—

Values of $\sqrt{\pi/\kappa}$ deduced from the Annual Wave.

Thermometers compared.	From diminution of amplitude.	From retardation of phase.
No. 5 and No. 4	0·1222	0·1202
,, 5 ,, 3	·1212	·1181
,, 5 ,, 2	·1208	·1181
,, 4 ,, 3	·1191	·1141
,, 4 ,, 2	·1193	·1159
,, 3 ,, 2	·1196	·1178
Means . . .	0·1204	0·1174
Mean of both . . . 0·1189		

The values deduced from the half-yearly wave in a similar way, are as follows :—

* 'Greenwich Observations,' 1860 (exciii).

Values of $\sqrt{\pi/\kappa}$ deduced from the Half-yearly Wave.

Thermometers compared.	From diminution of amplitude.	From retardation of phase.
No. 5 and No. 4	0·1445	0·1236
„ 5 „ „ 3	·1393	·1187
„ 5 „ „ 2	·1306	·1129
„ 4 „ „ 3	·1287	·1092
„ 4 „ „ 2	·1164	·1022
„ 3 „ „ 2	·1033	·0949
Means . . .	0·1271	0·1102
Mean of both . . . 0·1187		

I have omitted the results derived from the readings of No. 1, as they seem too much affected by short period variations to afford reliable results. This thermometer, too, is buried in a surface soil which is of quite a different character from the sandy gravel containing the other thermometers.

The values of $\sqrt{\pi/\kappa}$ deduced from the annual wave are, of course, much more trustworthy than those obtained from the half-yearly wave, and the larger discrepancies in the individual results from the latter are not surprising. It is, however, satisfactory to note how these corroborate the others, showing, for instance, larger values resulting from the comparison of Nos. 5 and 4 than from that of any other pair. This may possibly be due to a smaller value of κ for the stratum of gravel about 4 feet thick which separates these two thermometers, than for the higher strata, or to the fact of No. 5 being buried at some distance (9 feet 6 inches) from the vertical plane containing the other three.

Unfortunately, when the pits were open, no very critical examination of the character of the gravels at different depths was made; but it is proposed to repair this omission when next the thermometers are dug up.

The excessively close agreement of the mean values of $\sqrt{\pi/\kappa}$ derived from the annual and half-yearly waves is very remarkable (especially in view of the fact that the results are deduced from the observations of a single year), and seems to indicate a high degree of precision in the observations.

The systematically larger values found from the diminution of amplitude, as compared with those deduced from the retardation of phase must be traced to some other cause, and may possibly be due to the proximity of the Observatory building, the south front of which is situated at a distance of 36 feet from the thermometers. The temperature of the ground beneath the buildings would in all probability be different from that at an equal depth beneath the exposed surface. There would, therefore, be a transfer of heat which would render equation (*e*) no longer strictly applicable.

A solution on the hypothesis that the heat which is conducted in this way may be represented by $\mu\theta$ leads to the two values

$$\sqrt{\pi/\kappa} = 0{\cdot}1189 \text{ from the annual wave,}$$
$$\text{and } \sqrt{\pi/\kappa} = 0{\cdot}1184 \text{ from the half-yearly wave,}$$

a very satisfactory agreement; but the values found for μ/κ on the same hypothesis, viz. :—

$$\mu/\kappa = 0{\cdot}0007 \text{ from the annual wave,}$$
$$\text{and } \mu/\kappa = 0{\cdot}0080 \text{ from the half-yearly wave,}$$

differ too much to admit of any confidence in this additional term as representing the exchange of heat.

We cannot, however, be much in error in taking

$$0{\cdot}1188$$

as the value of $\sqrt{\pi/\kappa}$ for the gravel in which the thermometers are sunk.

The value of the same quantity as found by Professor EVERETT from the Greenwich Observations was

$$0{\cdot}09175,$$

and for the three stations at Edinburgh, from Professor FORBES' observations, Lord KELVIN obtained,

Calton Hill, trap rock	0·1154,
Experimental Garden, sand	·1098,
Craigleith Quarry, sandstone . . .	·06744.

From the equations (*f*) we find from each thermometer a value of the amplitude and of the retardation of phase of each wave at the surface.

Denoting the amplitude of the annual wave at the surface by P_{01}, we have

$$\log P_{01} = \log P_1 + \sqrt{\frac{\pi}{\kappa}} \,.\, x. \text{*}$$

Substituting the values of P_1 and x for each of the thermometers 2, 3, 4, and 5, we get four separate values of P_{01}. These are

Thermometer.	P_{01}.
	°
5 . . .	15·48
4 . . .	·70
3 . . .	·70
2 . . .	·72
Mean =	15·65 = amplitude of annual wave at surface.

* *Cf.* QUETELET, 'Annales de l'Observatoire Royal de Bruxelles,' tome iv., 1845, p. 110.

Similarly, for the half-yearly wave, we find

Thermometer.	P_{02}.
	°
5 . . .	2·60
4 . . .	3·01
3 . . .	3·09
2 . . .	2·97
	2·92 = amplitude of half-yearly wave at surface.

Putting these values in the first of equations (f) we can determine the value of x, corresponding to a given value of $P_{\prime\prime}$, or the depth at which the amplitude of the wave is reduced to any given value, on the hypothesis that the conditions prevailing above 10 feet remain unchanged at greater depths.

Although theoretically there is no invariable layer so long as equations (f) are applicable, still we may consider that an annual variation of 0°·02 F. is less than can be certainly detected. The stratum, therefore, at which the amplitude of the annual wave is reduced to 0°·01 may to all intents and purposes be considered as invariable.

For this depth we have M log $P_1 = -2$ (M being the modulus of common logarithms), and therefore

$$x = (2 + \log P_{01})/M\sqrt{\pi/\kappa}.$$

Thus we find as the depth at which the amplitude of the annual wave is reduced to 0°·01,

$$x_1 = 61\cdot21 \text{ French feet} = 66\cdot0 \text{ English feet},$$

and similarly for the half-yearly wave,

$$x_2 = 33\cdot78 \text{ French feet} = 36\cdot0 \text{ English feet}.$$

The depths at which the annual and half-yearly waves are reduced to an amplitude of 0°·1 F. are found in a similar way to be 45·3 and 21·4 English feet respectively.

This paper deals with the observations of a single year, and the results accordingly exhibit discrepancies between theory and observations which, although they are less than might have been *à priori* expected, are greater than one would like to see. These discrepancies are due partly to the fact that the temperature variations are not strictly of a periodic character as the theory supposes, and as such they might be expected to be diminished in the mean of a number of years, and partly to irregularities, physical and formal, in the surface.

The other source of irregularity considered by Lord KELVIN in his paper, referred to above, namely, thermometric errors arising from the uncertainty as to the temperature of the liquid in the long stems of the thermometers used in Professor

FORBES' observations, does not in this case apply, and if other errors peculiar to the platinum thermometers exist, they seem to be confined within much smaller limits.

In fact this mode of thermometry seems especially suited to the investigation of underground temperatures on account of its freedom from this source of error, and the convenience with which the observations may be made.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

DAILY READINGS OF UNDERGRO

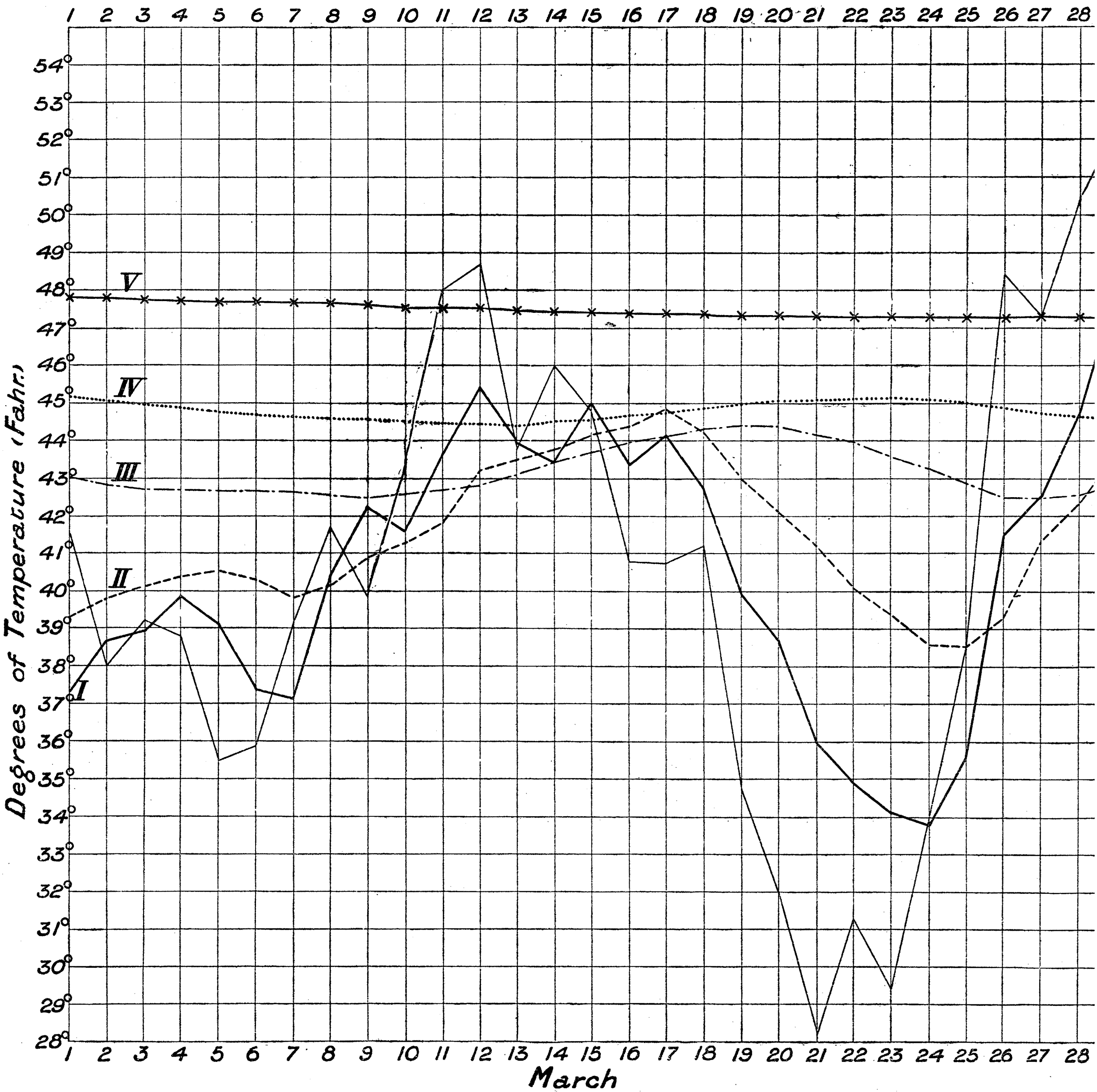
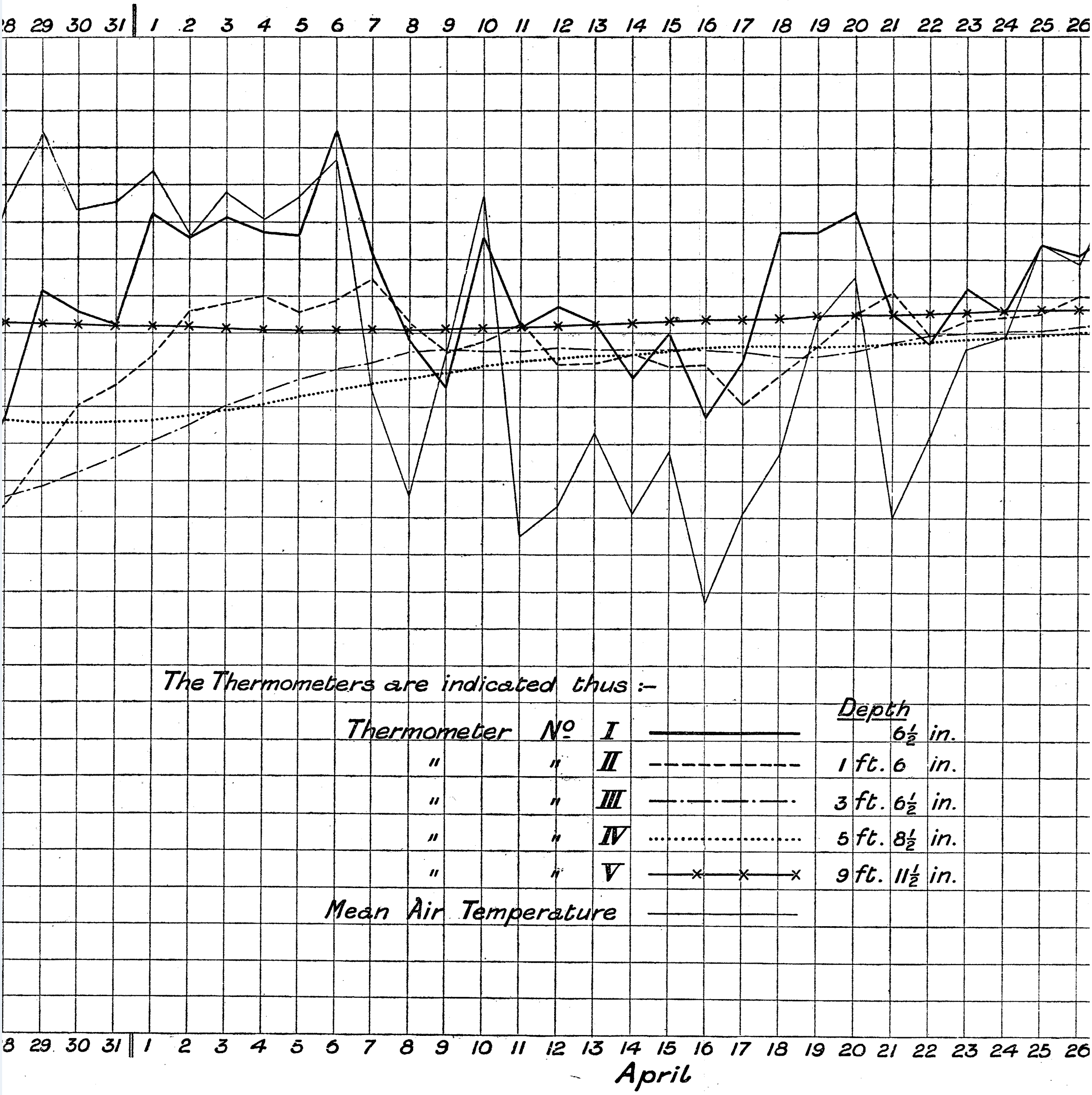


PLATE 1.

...ound Thermometers. March and April, 1899.



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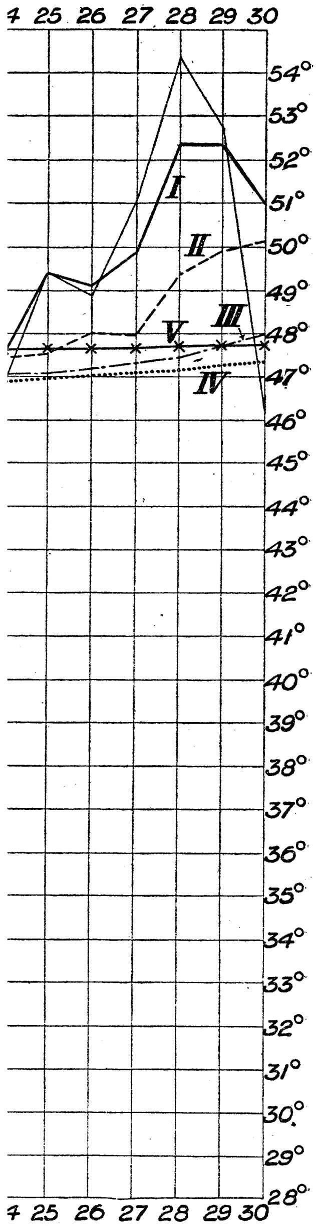
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES



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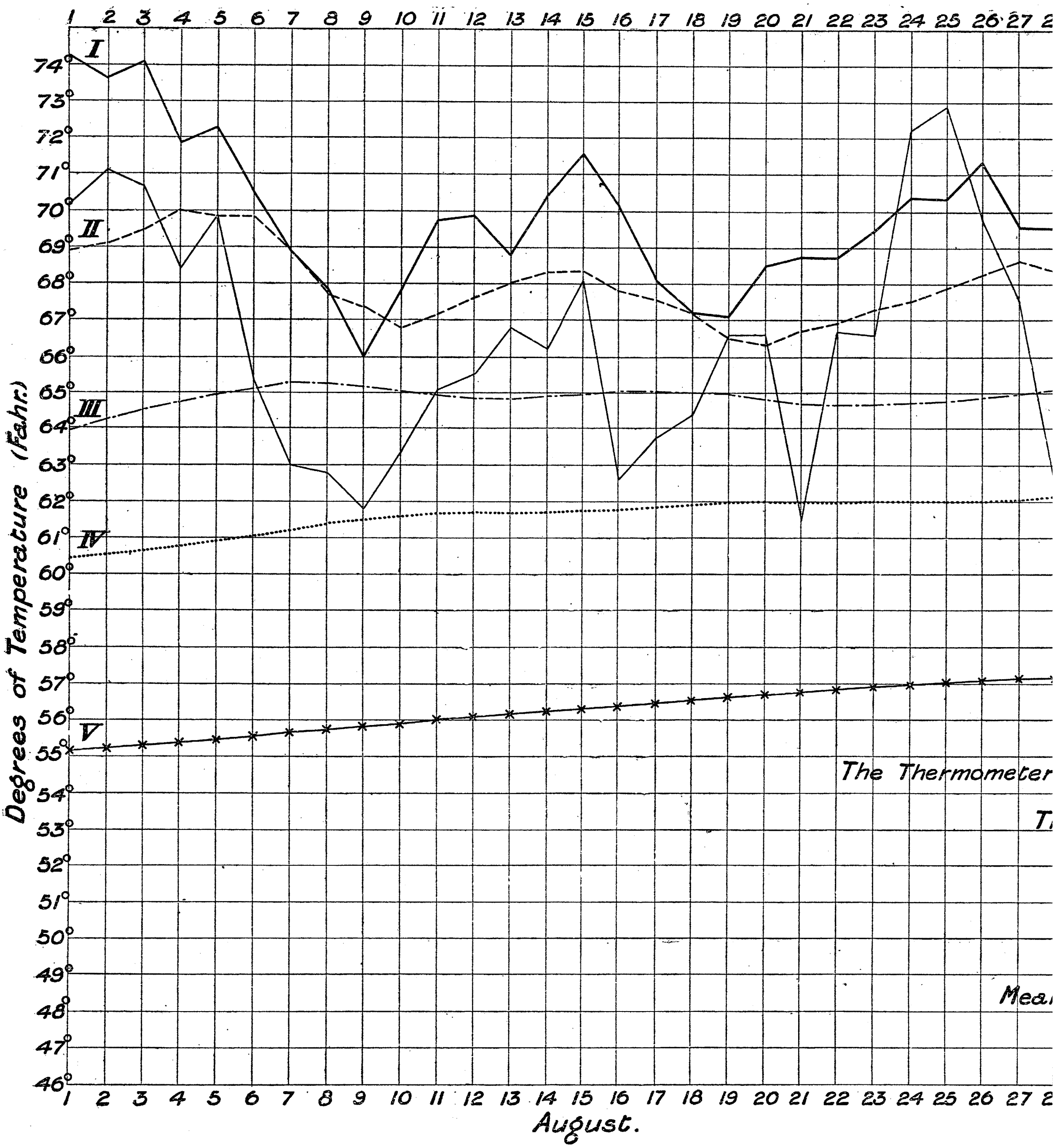
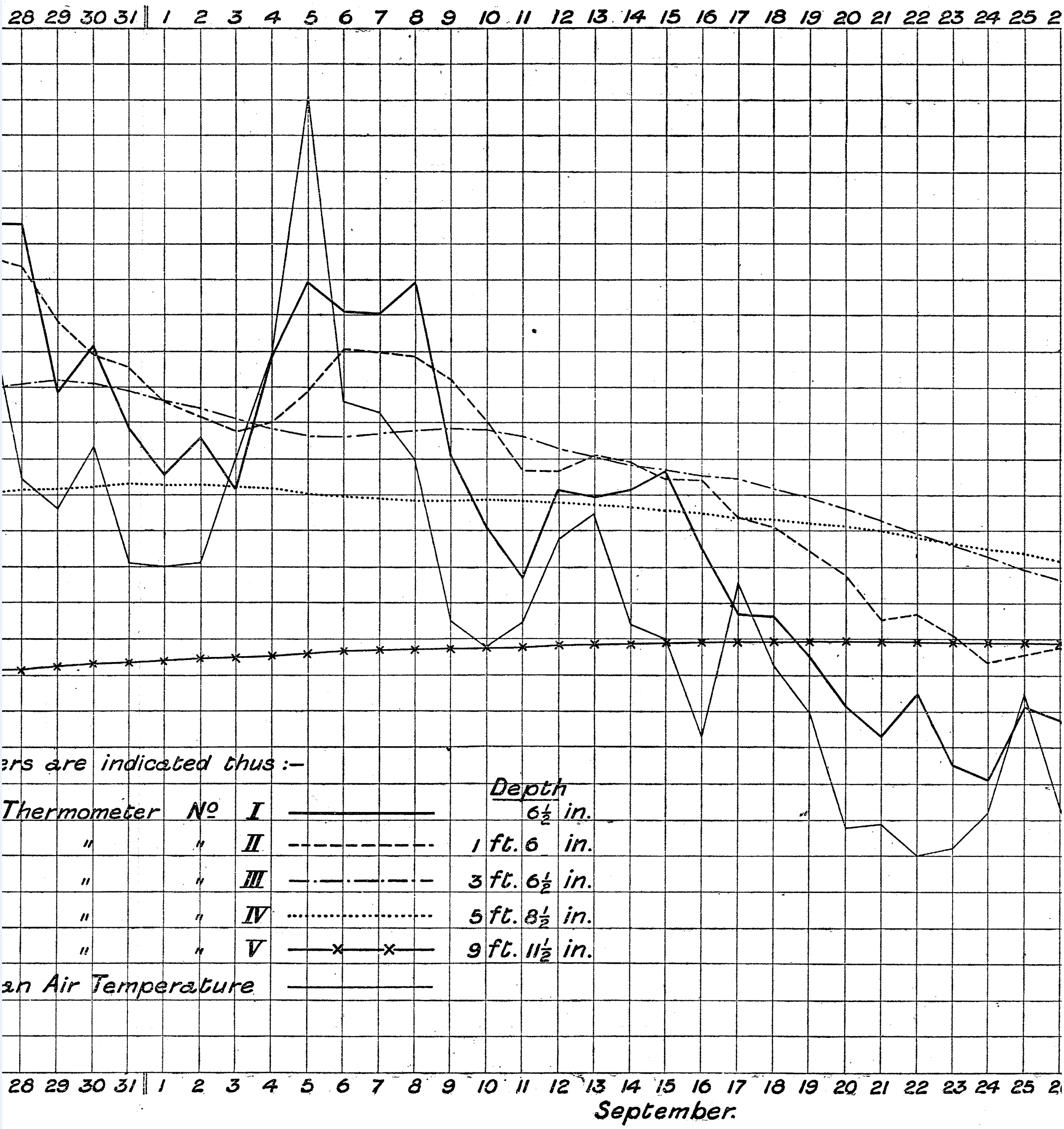


PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

PLATE 2.

UND THERMOMETERS. August and September, 1899.



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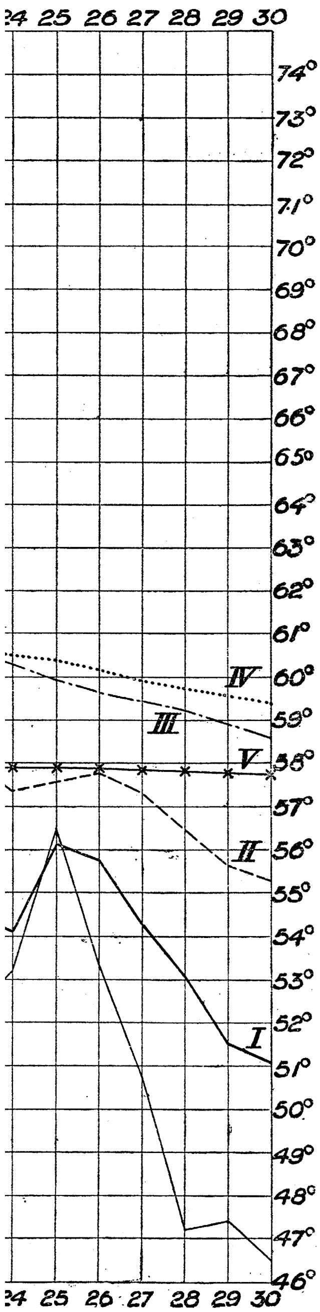
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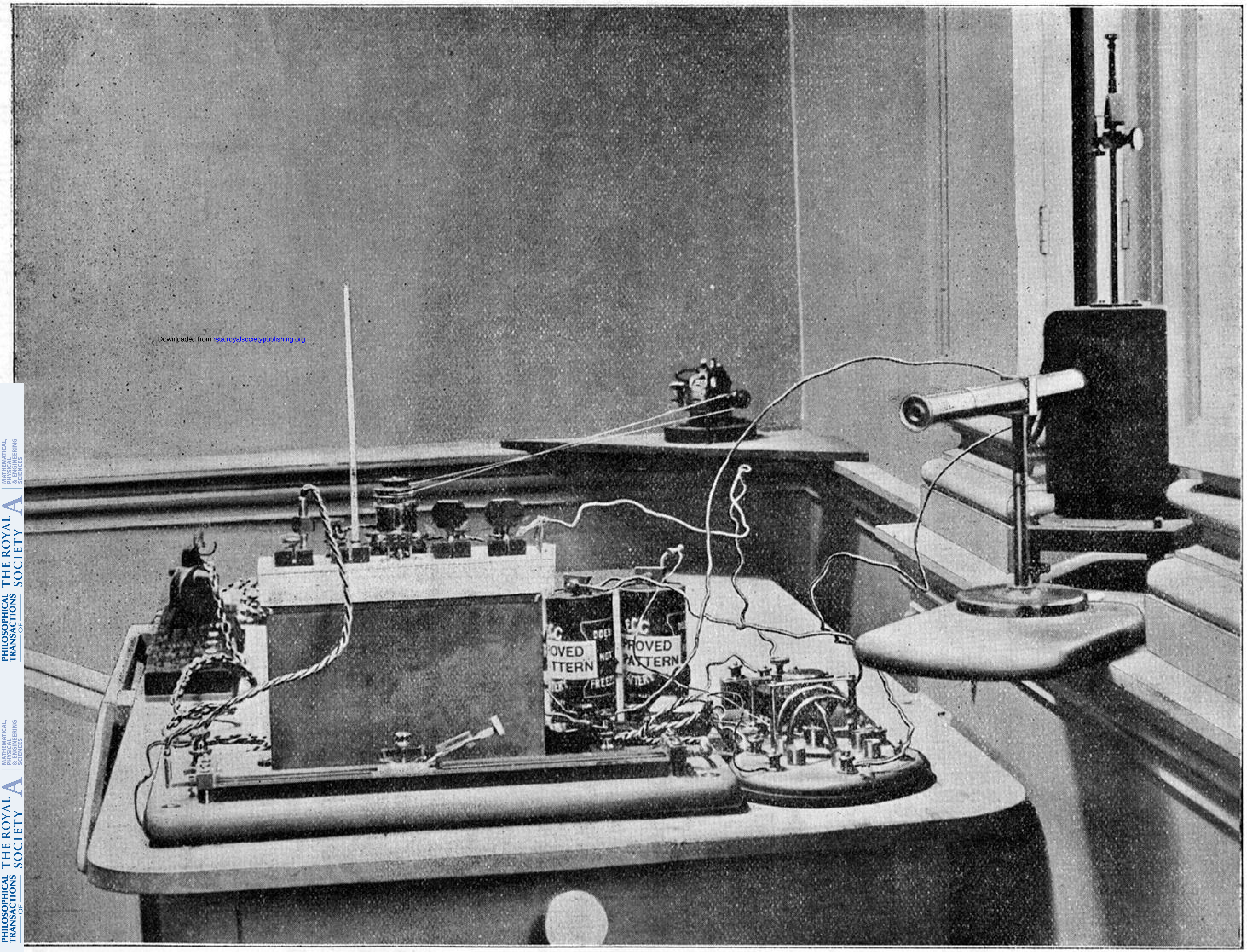

Fig. 1.—Resistance Box and Galvanometer at the Radcliffe Observatory, Oxford.

PLATE 1.

Daily Readings of Underground Thermometers. March and April, 1899.

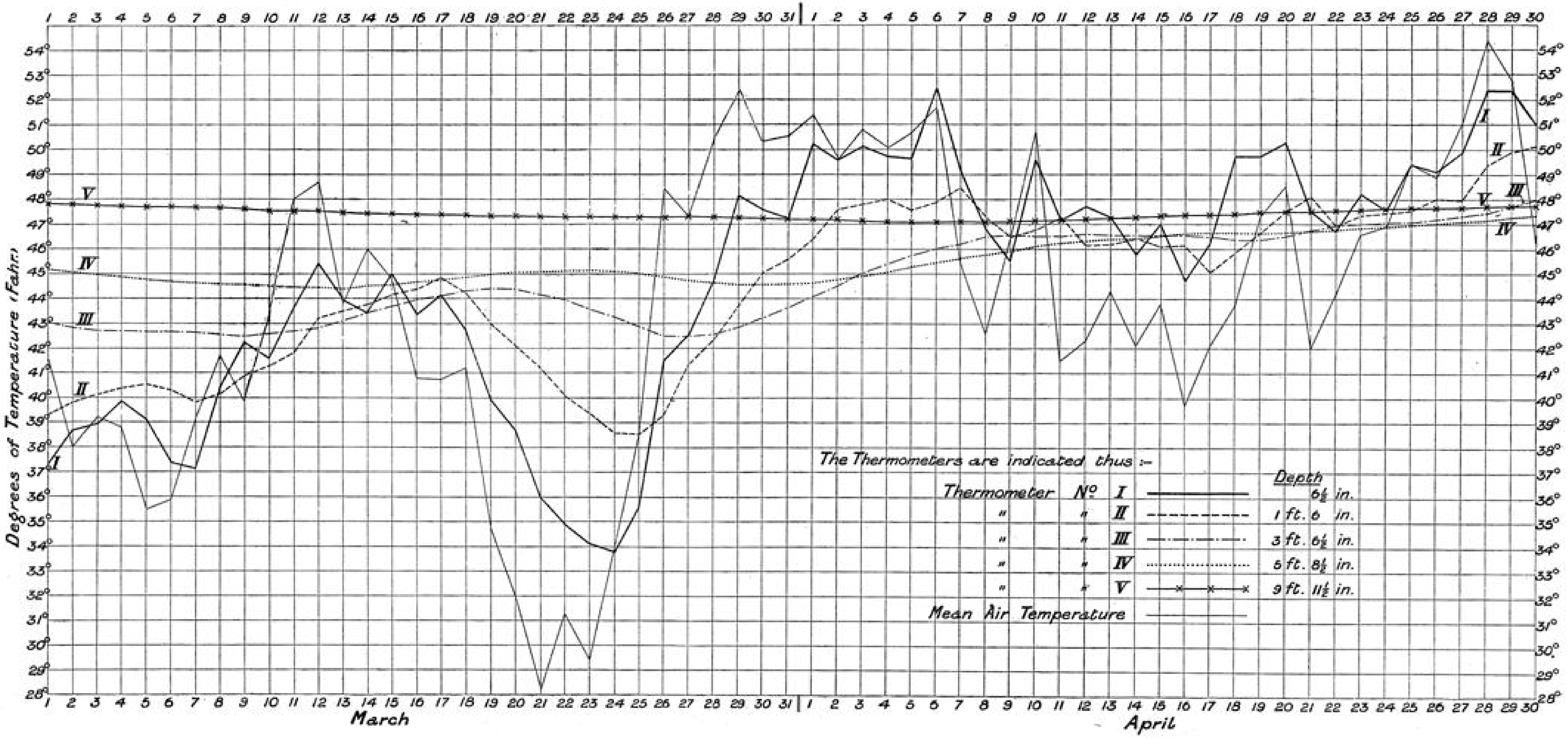

PLATE 2.

DAILY READINGS OF UNDERGROUND THERMOMETERS. August and September, 1899.